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Supersonic Laminar Boundary-Layer Separation by Slot Injection

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Solutions to the interacting laminar boundary-layer equations with surface injection are obtained for cases when classical theory predicts catastrophic blowoff. For the case of slot injection into a supersonic mainstream, implicit finite-difference solutions were obtained that show that interaction effects drastically alter the local pressure distribution and virtually eliminate the blowoff point. Favorable comparisons are made with asymptotic triple-deck solutions where appropriate. It is concluded that the present numerical technique works well, that the solutions honor triple-deck concepts, that blowoff does not occur in the injection region, and that separation will occur ahead of the slot, if at all.

Nomenclature

C	= Chapman-Rubesin constant, see Eq. (2e)
F	= normalized longitudinal velocity
$\dot{m}$	= nondimensional injection mass flow rate
M	= Mach number
n	= coordinate normal to surface
P	= triple-deck pressure correlation, see Eq. (14a)
p	= nondimensional pressure
Pr	= Prandtl number
Re	= Reynolds number
s	= longitudinal surface distance
T	= nondimensional static temperature
T_0	= nondimensional total temperature
u	= nondimensional longitudinal viscous velocity
U	= nondimensional longitudinal inviscid velocity
v	= nondimensional vertical velocity
V	= normalized vertical velocity
$\tilde{V}_w$	= triple-deck injection velocity, see Eq. (14b)
x	= nondimensional Cartesian coordinate
X	= triple-deck longitudinal coordinate, see Eq. (12)
β	= inviscid pressure gradient parameter
γ	= gas ratio of specific heats
δ	= Levy-Lees displacement thickness function
$\delta_{1,2}$	= displacement thickness functions, see Eq. (18b)
δ^*	= boundary-layer displacement thickness
Δ	= blown boundary-layer displacement height
Δ_i	= inviscid injection-layer thickness
Δt	= time step increment
η	= reduced normal distance
θ	= normalized static temperature
θ_T	= total deflection angle of displacement body
μ	= nondimensional viscosity coefficient
ξ	= reduced longitudinal distance
ρ	= nondimensional density

Subscripts

e	= inviscid edge properties
w	= wall properties
∞	= freestream properties
s,n,ξ,η	= derivative with respect to respective variable
sep	= separation point
wi	= weak interaction state
0	= start of injection slot
1	= end of injection slot

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Introduction

IT is known that small amounts of surface injection strongly influence the development of a boundary layer along a surface, and in particular can induce separation (referred to as blowoff) of the viscous region. This type of flow is particularly prevalent on ablating re-entry bodies but can also be encountered when injection is used to cool a surface or to control a separation bubble already induced by some other mechanism, such as a shock wave. The basic problem that occurs with such flows is that, apparently, once blowoff is encountered, the classical boundary-layer approach fails ~~(see Refs. 1 and 2)~~ and a major modification of the analytical model must be made in order to obtain a solution. At the other extreme, with regard to analytical simplicity, once the blowing reaches the massive blowing level and the viscous layer is lifted off the body, a mixing layer between the inviscid mainstream and the injected fluid layer prevails. For this case a new and quite tractable model of the flow can be formulated that holds promise for predicting the important fluid processes at hand (see Ref. 2 for further discussion of this approach). However, there exists a void in understanding the fluid dynamics of moderate injection rates, these being the ones that carry the problem of weak injection, incipient separation to that of incipient massive blowing. Stewartson[2] and Smith and Stewartson[3,4] have given considerable attention to this problem area, using an asymptotic multideck analysis of the flowfield. While much progress has been made, those authors present several important questions as yet unanswered and find it possible only to speculate on the anticipated results until more information becomes available for the problem. Stewartson[2] has recently presented an extensive review of this problem so in the present writing only those areas still found lacking will be discussed.

The first question arises when the case of weak blowing along a flat plate is studied. Here interaction of the boundary layer with the mainstream is reasonably assumed to be of secondary importance, and numerical solution of the classical boundary-layer equations has been presented for the case of constant blowing from a flat plate by Catherall et al.[1] Most significantly, it was found that under the action of a constant injection velocity, boundary-layer separation (or blowoff) was predicted to always occur somewhere on the plate surface. Since a singularity was observed in the displacement thickness growth rate at the blowoff point, it is obvious that interaction effects must be accounted for in this region before the true picture of the phenomenon can be obtained. The fundamental unanswered question is whether or not the boundary layer will really blow off once interaction effects are properly accounted for. Stewartson[2] speculates that it will not occur but points to a severe lack of knowledge about this problem which hampers full understandiang of the phenomenon.

For the problem of moderate to strong injection through a slot on a plate, Smith and Stewartson[4] found that a consistent model of the flowfield could be formulated using the triple-deck concepts† (see also Ref. 2). They find that by accounting for interaction effects, wherein the injection region induces a region of compressive free interaction immediately ahead of the onset of injection, they can apparently capture the principle aspects of the problem. Unfortunately, the solution of the resulting set of equations was found to be quite tedious and critically tied to the manner in which the required downstream compatibility conditions were satisfied. Some solutions were presented for blowing levels that bordered on incipient separation conditions and these solutions provide some insight into the expected stronger blowing behavior. One case, apparently not fully numerically converged,[2] in which separation was observed is reported in Ref. 4 for the case where small blowing takes place over a long distance. With these limited number of cases the authors were able to speculate on the expected flow behavior when blowing becomes large enough to induce separation or blowoff. However, there remains the problem of actually bridging the gap from these nearly separated cases to the fully separated regime indicative of massive blowoff conditions. The main stumbling block appears to be the development of an effective means of solving those governing equations identified in Refs. 2-4 as appropriate to the problem. Since recent developments in interacting boundary-layer theory[5] indicate that such a method is at hand, it now appears appropriate to apply these concepts to the intermediate blowing problems.

The goal of the present effort is to contribute to clarification of some of the fundamental questions just discussed for weak to moderate slot blowing levels that are expected to involve separation. The approach taken follows closely that used by Werle and Vatsa,[5] wherein the interacting boundary-layer equations (which contain all the structure of the asymptotic multideck models) are solved using an alternating-direction implicit finite-difference technique that allows full and direct accounting of a downstream boundary condition on the problem. This technique is applied to the case of blowing into a laminar boundary layer on a $M_\infty = 4$ flat plate with both uniform and nonuniform blowing distributions examined. Solutions are obtained for blowing rates ranging from the weak, nondisruptive regime, all the way to the incipient massive blowoff regime with several cases containing regions of observed separated flow. In all slot injection cases studied it was found that when separation was observed, it always occurred ahead of the slot after a "free interaction" type compression region. It was also found in the cases studied that even when noninteracting boundary-layer theory predicted separation in the blowing region, interaction effects voided this result and ultimately moved the separation point out ahead of the blowing slot.

Governing Equations

The basic assumption is made that the interacting boundary-layer equations are appropriate for the present investigation of the mass injection regime. These equations are merely the classical boundary-layer equations with account made of their self-induced pressure gradients by assuming that the inviscid flow effectively "sees" the "displacement body" surface formed by adding the total displacement height (due to boundary-layer growth plus injected mass) to the original surface geometry. The validity of this approach is well established for the weak interaction region where it is clear that the approach is equivalent to Van Dyke's[6] higher order boundary-layer theory (see also Ref. 7). Note that wall injection produces no formal change in the structure of the higher order theory. If and when separation (or in this case, blowoff) occurs, this approach is open to question and it is suspected that additional terms must be retained from the Navier-Stokes equations in order to properly model the resulting flow. However, recent developments stemming from triple-deck asymptotic analysis of separation problems[8-12] indicate that no such new terms are needed to model the flow, and that the interacting boundary-layer equations represent a composite set of equations that are uniformly valid to first order in the two principle decks of the viscous region (see Refs. 11 and 12 for further discussion of this point). These equations provide a basis for bridging the present gap between the weak and massive blowing regimes.

With this point aside, the governing equations are now presented in terms of Levy-Lees variables and the dependent variables (see Ref. 5 for more discussion of the nondimensionalization scheme)

$$F = u/U_e, \quad \theta = T/T_e, \quad V = [\eta_s F + \rho v \sqrt{Re}/\sqrt{2\xi}](2\xi)/\xi_s \quad (1a)$$

where

$$\xi = \int_0^s \rho_e \mu_e U_e \mathrm{d}s, \quad \eta = U_e \sqrt{Re}/\sqrt{2\xi} \int_0^n \rho \mathrm{d}n \quad (1b)$$

The governing equations with the assumption of a linear viscosity law now become

Continuity:

$$V_\eta + F + 2\xi F_\xi = 0 \quad (2a)$$

Momentum:

$$F_{\eta\eta} - VF_\eta + \beta(\theta - F^2) - 2\xi F F_\xi = 0 \quad (2b)$$

Energy:

$$\frac{1}{Pr}\theta_{\eta\eta} - V\theta_\eta + \frac{U_e^2}{T_e}F_\eta^2 - 2\xi F\theta_\xi = 0 \quad (2c)$$

where

$$\beta = \frac{2\xi}{U_e}\frac{\mathrm{d}U_e}{\mathrm{d}\xi} \quad (2d)$$

and, for a linear viscosity law,

$$\mu_e = C\mu_\infty \quad (2e)$$

where C is the usual Chapman constant.

The applicable boundary conditions are

$$F(\xi,\eta) \text{ and } \theta(\xi,\eta) \to 1 \text{ as } \eta \to \infty \quad (3a)$$

and

$$F(\xi,0) = 0, \quad V(\xi,0) = V_w(\xi), \quad \theta(\xi,0) = T_w/T_e \quad (3b)$$

with T_w assumed constant.

The wall velocity V_w is related to the physical wall velocity through Eq. (1a) as

$$V_w = \sqrt{Re}\dot{m}\sqrt{2\xi}/\rho_e \mu_e U_e \quad (4a)$$

†Note that use of the terms "strong," "moderate," and "slot" imply very definite meanings in the asymptotic formulation of the problem which are not completely adhered to in this descriptive context.

where $\dot{m}$ is the nondimensional mass flux rate across the wall and is given as

$$\dot{m}=\rho_w v_w \tag{4b}$$

Of importance here is the necessary modification to the usual displacement thickness definition in order to properly account for injection effects. This issue was clarified fully by Burggraf[13] and was resolved by defining the displacement thickness as that height above the surface at which the inviscid streamfunction vanishes (to second order). With this height defined as Δ, Burggraf obtains the relation

$$\Delta=\delta^*+\Delta_i \tag{5a}$$

where δ^* is the classical displacement thickness term given as

$$\delta^*=\sqrt{2\xi}\delta/\rho_e U_e\sqrt{Re} \tag{5b}$$

$$\delta=\int_0^\infty(\theta-F)\mathrm{d}\eta \tag{5c}$$

where Δ_i is the injection surface height given as

$$\Delta_i=\frac{1}{\rho_e U_e}\int_{s_0}^{s}\dot{m}\mathrm{d}s \tag{5d}$$

or

$$\Delta_i=\frac{1}{\rho_e U_e\sqrt{Re}}\int_{\xi_0}^{\xi}\frac{V_w}{\sqrt{2\xi}}\mathrm{d}\xi \tag{5e}$$

and s_0 is the point at which injection begins.

Before considering the interaction law that couples the local pressure gradient parameter β to the displacement body growth Δ, it is instructive to first consider the classical case of a noninteracting boundary layer. For the case of flat plate blowing this is recovered simply by setting $\beta=0$ in Eq. (2b) and taking all inviscid values at their undisturbed freestream levels. The resulting equations are still of the nonsimilar partial differential type and can be solved using the implicit finite-difference techniques deveoped by Blottner[14] and many others. Initial conditions for these solutions can be recovered from the self-similar family of boundary-layer solutions. This family of solutions is recovered from the flat plate versions of Eqs. (2) and (3) which reduce to ordinary differential equations under the additional restriction that V_w be a constant from the leading edge up to the point of interest. Viewed in another way, the general solution to the governing equations will be of the self-similar type over the region in which V_w is held fixed.

The physical wall injection velocity can be written from Eq. (4a) using a linear viscosity law as

$$v_w=V_w(T_\infty/T_w)\sqrt{\frac{C}{sRe_\infty}} \tag{6}$$

so that $V_w=\text{const}$ implies the physical injection velocity decays proportional to $s^{-1/2}$ and from Eq. (5), the displacement body growth can be described by

$$\Delta=\sqrt{\frac{2sC}{Re_\infty}}(\delta+V_w) \tag{7}$$

where δ can be computed in a straightforward manner from the momentum and energy equations. For the case of a unit Prandtl number, the energy equation can be eliminated to give

$$\delta=T_w\delta_1-\frac{\gamma}{2}M_\infty^2\delta_2=\frac{T_w}{T_0}\left(1+\frac{\gamma-1}{2}M_\infty^2\right)\delta_1-\frac{\gamma}{2}M_\infty^2\delta_2 \tag{8a}$$

where δ_1 and δ_2 are universal functions dependent only on V_w and are given as the familiar integrals

$$\delta_1=\int_0^\infty(1-F)\mathrm{d}\eta,\quad \delta_2=\int_0^\infty F(1-F)\mathrm{d}\eta \tag{8b}$$

Equation (7) will play a fundamental role in the formulation of the injection problem when there is to be a region of injection ahead of the initialization point.

For the case where a region of constant injection velocity v_w is initiated at some point s_0 from the leading-edge of a plate, the displacement body is given directly by Eq. (5d) as

$$\Delta=\sqrt{\frac{2Cs}{Re_\infty}}\delta+\frac{T_\infty}{T_w}V_w(s-s_0) \tag{9}$$

where δ is no longer a constant but must be obtained from a numerical solution of the flat plate version of Eqs. (2) and (3). This case will be considered in more detail later in this paper.

To account for the effects of the boundary-layer growth interacting with a supersonic mainstream, it is only necessary to couple the inviscid edge flow properties to the displacement body growth. This is most easily and reliably accomplished (see Ref. 5) using the generalized supersonic/hypersonic tangent-wedge pressure law to relate flow deflection to edge pressure through the expression[15]

$$p/p_\infty=\frac{1}{\gamma M_\infty^2}+\frac{\gamma+1}{4}\theta_T^2\left[1+\sqrt{\frac{1}{1+(\gamma+1/4)^2(M_\infty^2-1)\theta_T^2}}\right] \tag{10a}$$

where, for injection along a flat plate,

$$\theta_T=\tan^{-1}\left(\frac{\mathrm{d}\Delta}{\mathrm{d}x}\right)\approx\frac{\mathrm{d}\Delta}{\mathrm{d}x} \tag{10b}$$

The remaining flow variables (U_e,μ_e,ρ_e,T_e) are then determined from the isentropic flow relations. Using Euler's equation, the pressure gradient parameter β of the momentum equation is then related to the second derivative of Δ (see Refs. 5 or 16). The most significant aspect of this last result is that it produces a major modification of the nature of the governing equations that has a drastic effect on the method of solution. As discussed by Werle and Vatsa,[5] Stewartson,[2] Smith and Stewartson,[4] and others, the interaction problem is of a boundary-value type and requires statement of some downstream property for closure on the problem. Note that this need for a downstream boundary condition has nothing to do with separated flow theory, but stems directly from the coupling of an inviscid and viscous region. In fact Smith and Stewartson[4] had considerable difficulty in solving the nonseparated but interacting injection problem using a forward marching numerical technique apparently due to this point. In their approach, the boundary-value nature of the problem was accounted for by iterating on conditions at some upstream station in order to reasonably achieve the downstream solution behavior. This difficulty is identical to that encountered for interacting flows over flat plate compression ramps and has been very effectively overcome by Werle and Vatsa[5,16] using a numerical method that directly imposes a downstream condition during all stages of the solution process.

It is only necessary then to determine which downstream condition is the most appropriate for the case at hand. Since the requirement for such a condition comes from the coupling with the inviscid flow it should suffice to apply an inviscid flow property at some downstream extremity of the flow region of interest. Stewartson[2] discusses the appropriateness of this approach and concludes that for the flat plate, $p\to p_{\text{inv}}$ as $s\to\infty$. Since this condition can only be met in an asymptotic sense, it is expected that it will only be met in an approximate fashion, the influence of which must be assessed a posteriori.

Numerical Method

The solution to the governing equations was obtained using the finite-difference technique presented in Refs. 15 and 17 for the similar problem of ramp-induced boundary-layer separations. The only modifications made to the basic method were of rather minor consequence. The first change was the addition of a nonzero velocity normal to the wall in the solution of the continuity equation, which caused no new complications. The numerical scheme was further simplified by removing one of the time multiplier functions h^* [of Eq. (10), Ref. 5]. In addition, the displacement thickness terms and their derivatives appearing in the momentum equation (see Ref. 5) were reinterpreted to be Δ, the total displacement height. Finally, the present approach requires statement of the pressure level that the interaction equations must satisfy far aft on the plate (Ref. 5 imposed a zero pressure gradient requirement aft of the region of strong interaction). In the present approach the pressure at the last station in the mesh is required to be its inviscid value as obtained from Eqs. (10a) and (10b) by replacing Δ with the injection thickness alone, Δ_i.

The finite-difference mesh chosen extends from $s=0.4$ to $s=2.2$ in increments of $\Delta s=0.02$. Across the viscous region 40 mesh points were employed with a value of $\Delta\eta=0.4$. Previous step-size studies and an error analysis conducted for the same flow property state with a ramp-induced separation bubble[17] indicated the expected error in the surface pressure distribution to be of the order of 1% or less while the surface skin friction might vary upwards of 5%. These limits were considered acceptable for present interest and are not believed to detract from the validity of the final results. A further reduction in mesh size will increase the accuracy of the solution at the expense of increased computer time. Other means of increasing accuracy without increasing computing time such as using a Crank-Nicolson finite-difference scheme,[14] quasilinearization techniques,[18] or spline-fitting approaches[19] should benefit the present solution and should be considered in future applications if computing speed is an issue.

In the present approach, the timelike relaxation scheme time step Δt employed for ramp-induced separation (see Ref. 17) was carried over to the injection case with no attempt to iterate on its value to obtain an optimum convergence rate to the "steady-state" solution. The value used here was $\Delta t=0.25$, and the procedure followed was to study the injection influence at various stages v_w, letting the value build up from zero to a final value that ultimately placed the separation point close to the front end of the finite-difference mesh at $s=0.4$. Thus, for example, the interacting equations were first solved with a small injection rate and allowed to converge in time (on the order of 10 time steps or 1 min of computer time on an IBM 370-168 computer). A step increase in v_w was then activated to send the solution to its next level of interest. The total time consumed for the family of solutions considered was on the order of 5 to 10 min depending on the number of v_w cases studied.

To obtain solutions to the noninteracting boundary-layer equations, the identical numerical scheme described above was employed with the interaction effects shut off. In particular, the inviscid properties were stated entirely in terms of the known undisturbed properties, thereby eliminating the need for the timelike relaxation approach. For the noninteracting case a single integration sweep along the length of the plate is all that is required for a converged solution. This was accomplished in about 6 s of computer time for the identical mesh used in the interaction studies. Here, however, a new subtle difficulty emerges at the start of injection that requires attention. For the noninteracting case it is clear from Eq. (10) that the inviscid flow is modified by injection in that, in accord with airfoil theory, the slope of the "effective" body is changed with the injection rate. In particular, a constant rate of injection produces a wedge-shaped "effective" body at the angle of

$$\frac{d\Delta_i}{dx}=\frac{T_\infty}{T_w}v_w \tag{11}$$

to the mainstream. Thus the problem encountered with a discontinuous blowing distribution at some point on a flat plate is identical to that encountered at a compression ramp juncture point, i.e., an infinitely adverse pressure gradient. In the case of a real flow, both the combination of interaction effects on the mainstream and an expected local reduction in v_w due to interaction would serve to eliminate this problem. However, if one insists on solving the pure noninteracting model some artifice must be employed to pass over the singularity at the junction point. Since for the weak blowing cases the injection-induced pressure level is necessarily small if boundary-layer concepts are to be honored [i.e., $v_w=0(Re^{-1/2})$], it seems realistic to ignore the pressure rise altogether and let the injection take place in a mainstream that was uniform over the entire plate length. Such an approximation becomes less acceptable as the blowing rate (and thus downstream pressure level) increases, but by then the infamous blowoff phenomenon sets in and recourse to the interacting boundary-layer equations must be made to obtain meaningful results.

Results and Discussion

For all of the cases studied, the base flow was taken as that over a flat plate at $M_\infty=4.0$, $Re=6.8\times10^4$, and $T_w/T_0=1.0$, corresponding identically to the experimental flow conditions used by Lewis et al.[20] to study boundary-layer separation induced by compression ramps placed at $s=1.0$ from the leading edge. This case was chosen for two specific reasons. First, it represents a set of flow conditions for which a true laminar state is known to exist even though the flow is separated by as much as a 10 deg compression ramp.[20] The second reason involves accuracy considerations, it being necessary to have some basis for accepting the quantitative and qualitative value of the results obtained. This is by no means a secondary issue for as clearly shown by asymptotic studies of separated flows[8-12] there are very fine flow structures that establish the entire interacting mechanism which must be resolved accurately by the finite-difference mesh in order to capture the true solution of the differential equations. Since the base flow case used here has already been

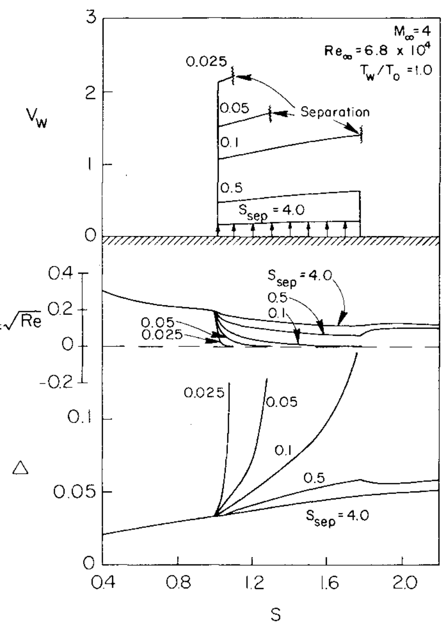

Fig. 1 Noninteracting solutions with uniform slot injection.

Table 1 Slot injection study parameters

X_l	s_l	s_{sep}	v_w	$\tilde{V}_w$	$\dot{m}$	p_0	p_{wi_0}	P_0
5.0	1.8	4.0	0.001656	0.9244	0.000344	1.129	1.101	0.067
		2.0	0.002341	0.13073	0.000557	1.142		0.097
		1.0	0.003311	0.18488	0.000788	1.166		0.158
		0.5	0.004683	0.26147	0.001114	1.201		0.246
		0.1	0.010471	0.58466	0.002493	1.403		0.743
		0.05	0.014805	0.82683	0.003526	1.560		1.129
		0.025	0.020942	1.16932	0.004986	1.710		1.498
		0.0125	0.029617	1.65365	0.007052	1.918		2.011

well studied numerically[5,17,21] and the accuracy level well established, it was again employed here to lend credence to the results.

The first blowing configuration studied was that of slot injection initialized at $s_0 = 1.0$ and continued to s_l. Downstream of s_l, zero injection was specified to the end of the finite-difference mesh at $s = 2.2$. The blowing rates and distance s_l were chosen to allow a reasonable correspondence with the previous injection studies.[1-4] Of particular interest were the noninteracting plate injection studies of Catherall et al.[1] and the triple-deck asymptotic analysis of slot injection with interaction due to Smith and Stewartson.[4] The injection length was chosen to correspond exactly to the case of $X_l = 5$ descibed in Ref. 2, where X_l is the asymptotic inner-deck longitudinal variable and is related to the physical slot length through the relation

$$s_l - s_0 = \frac{(T_w/T_\infty)^{3/2}}{(0.332)^{5/4}} \left[\frac{C}{Re_\infty (M_\infty^2 - 1)} \right]^{3/8} X_l \qquad (12)$$

which for the present case gives $s_l = 1.8$.

The injection velocities were varied from zero to values large enough to induce incipient blowoff of the boundary layer. This was accomplished through variation of the parameter s_{sep}, identified as the point at which the noninteracting boundary-layer solution of Ref. 1 predicts separation for uniform plate-type blowing starting at the leading edge. The results from this investigation provide the wall velocity needed for each s_{sep} as

$$v_w = 0.8635/\sqrt{s_{sep} Re_\infty} \qquad (13)$$

Thus the blowing rate can be systematically varied to move the noninteracting separation point at will, thereby allowing a view of the influence of interaction effects on the resulting flow structure.

For comparison purposes the noninteracting boundary-layer solutions are first shown in Fig. 1. The parametric dependence on s_{sep} is straightforwardly related to the wall velocity v_w, and the mass flux ratio $\dot{m}$, with the resulting values given in Table 1. The resulting distribution of the transformed wall velocities V_w are also shown in Fig. 1, these being merely a family of parabolas starting at $s = 1.02$. As seen in the figure, decreasing s_{sep} below 0.50 induces separation in the blowing region for all subsequent cases. While this might be interpreted in terms of the blowoff phenomenon, note must first be made of the displacement thickness shapes in this region. As pointed out by Catherall et al.,[1] this discontinuous growth rate would surely induce large enough deflections to the inviscid stream to nullify the zero interaction assumption of these solutions. Thus whether or not blowoff will occur for these cases can only be answered with use of the interacting boundary-layer approach.

The results obtained by adding interaction effects to the constant injection velocity case are shown in Fig. 2 for the same values of s_{sep}. Attention is first drawn to the surface pressure distributions of this figure, where it should be noted that even for no injection, the weak interaction square-root decay of pressure is of an appreciable level for the present flow configuration. The weak blowing case of $s_{sep} = 4.0$ is seen to only slightly modify this in the injection region with a slight increase of pressure over the blowing region and a quick recovery to the downstream boundary condition that $p \rightarrow p_\infty$. In fact it appears from this figure that the imposition of $p = p_\infty$ at $s = 2.2$ is a little premature for this particular flow regime and that some sort of weak interaction pressure decay would be more appropriate. This issue will be probed later in this section but the main issue here is that the present numerical method has no difficulty accounting for a downstream boundary condition (as have earlier studies) and it remains now to adjust that condition to the most realistic level. For present interest, simply moving the position of the downstream condition aft of $s = 2.2$ would serve such interest but this was not carried out here. Attention here was directed toward the upstream separation region and thus this compromise of the accuracy due to this approximation at the downstream boundary was deemed acceptable.

Figure 2 shows a marked influence of the wall injection on the flowfield in somewhat the same manner as that observed in the noninteracting case. As s_{sep} is decreased (causing an increase in v_w), the displacement thickness grows, and the shear level in the blowing region drops. Here, however, the surface pressure level is markedly increased over its weak interaction level and starts to demonstrate a free interaction region ahead of the slot in much the same manner as that predicted by Smith and Stewartson[4] for the nonseparating cases. A comparison of the present results at the start of blowing with the results predicted for slot-type blowing by the analysis of Ref. 4 is shown in Fig. 3. Here the dependence of the injection-point pressure rise on the wall velocity is shown in terms of the inner-layer variables P_0 and V_w where these variables are related to the physical variables through the relations‡

$$P_0 = (0.332)^{3/4} (p_0/p_{wi}) [Re_\infty (M_\infty^2 - 1)/C]^{1/4} \qquad (14a)$$

$$\tilde{V}_w = v_w (T_\infty/T_w)^{1/2} Re_\infty^{3/8} / [(0.332)^{1/4} C^{3/8} (M_\infty^2 - 1)^{1/8}] \qquad (14b)$$

Smith and Stewartson[4] were only able to obtain solutions without separation ahead of the slot. In this range they gave a linearized solution valid for small V_w, and solved the full nonlinear governing equations for one V_w for this configuration. Comparison of the present results for small V_w is quite good, with the origin of the difference observed between the two cases most probably due to the fact that the present case was performed at a relatively low finite Re_∞ while the asymptotic theory is strictly valid only as $Re_\infty \rightarrow \infty$. Note that as V_w increases, the nonlinear nature of the solution becomes apparent, P_0 increasing above its low blowing asymptote in accord with that predicted by the nonlinear solution of Ref. 4. As V_w is increased to the point where separation is induced, the nature of the curve again changes as the rate of increae of P_0 diminishes. However, the highest blowing rate solution has to be interpreted carefully, for it is very likely that this case is being significantly influenced by the presence of a strong

‡For comparison purposes, the definition of P_0 used here was modified slightly over that used in Ref. 4 to account for the weak interaction pressure rise at this finite Reynolds number.

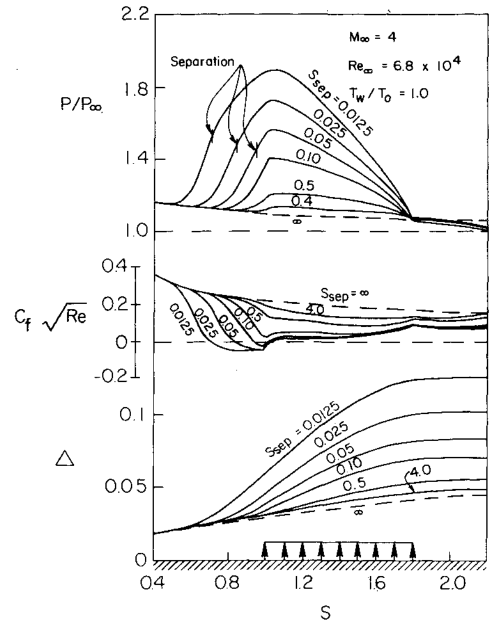

Fig. 2 Interacting solutions with uniform slot injection.

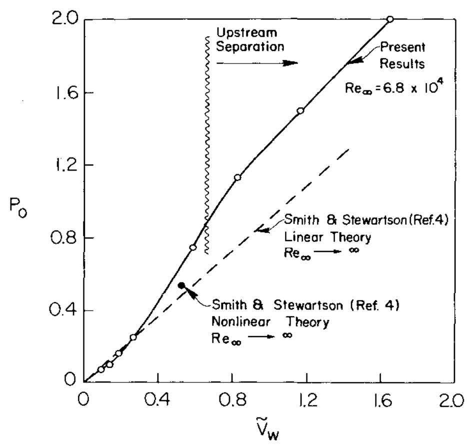

Fig. 3 Comparison of interacting boundary-layer and triple-deck solutions.

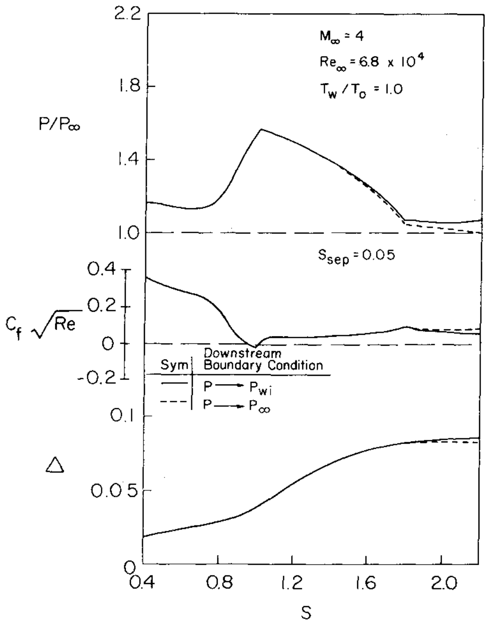

Fig. 4 Downstream boundary-condition influence.

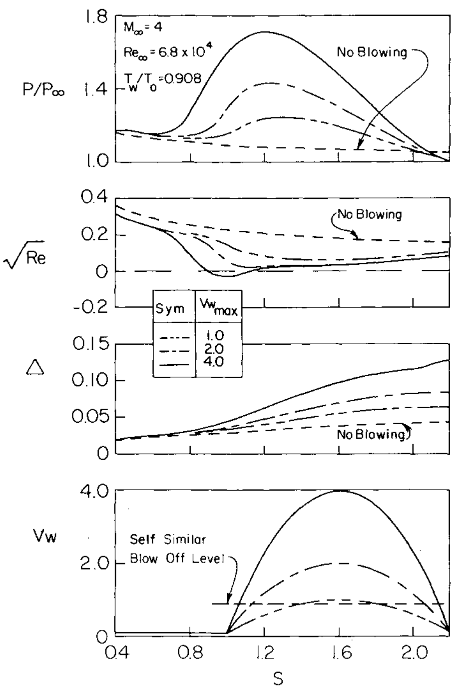

Fig. 5 Interacting solutions with nonuniform slot injection.

boundary-condition statement at $s = 2.2$. This is especially apparent when one considers the fact that all previous studies of free interaction regions (see Ref. 2) indicate that one should anticipate seeing a plateau in the pressure distribution at a level of $P_0 = 1.8$ before it reaches the peak level of 2 shown in Fig. 3. No such plateau is observed in Fig. 2 and thus the highest level of P_0 shown in Fig. 3 is suspect.

Nevertheless, the family of solutions shown in Fig. 2 clearly provides some insight into the nature of the interaction effects on flow properties. In particular, the interaction effect is seen to move the formal blowoff position always forward of the blowing region somewhat in accord with the conjecture of Stewartson et al.[2,4] Apparently, before the shear level in the blowing region drops to zero, the boundary layer ahead of this region reacts violently and lifts off the surface ahead of injection and is carried up and over the injectant. From these results it seems apparent that with an $\dot{m}$ of the order of 1%, one can anticipate the boundary layer to be fully blown off considerably far ahead of the injection port. For $\dot{m}$ values much larger than this, the whole question of the validity of boundary-layer concepts reemerges and it would seem more appropriate to adopt the massive blowing-type models of Cole and Aroesty[22] for such cases.

There remains a question concerning the influence of the downstream boundary condition on the numerical results. In particular, Fig. 2 shows that the imposition of the condition $p = p_\infty$ at $s = 2.2$ for the blowing cases produces an inconsistency between the low-blowing ($s_{sep} = 4$) and nonblowing ($s_{sep} = \infty$) pressure distributions aft of $s = 1.8$. The no-blowing results were obtained from weak interaction theory and indicate that taking $p = p_\infty$ at $s = 2.2$ would have induced up to a 6% error in pressure. A numerical experiment was conducted to investigate the spread of this error and its results are shown in Fig. 4. Here the solution for $s_{sep} = 0.05$ obtained by taking the downstream pressure level to be the weak interaction pressure p_{wi} at $s = 2.2$ is compared to its counterpart of Fig. 3. For the skin friction and displacement thickness the error is apparently very localized, whereas for the pressure its effect spreads slightly forward. This localization of the exit plane error apparently stems from the basic "stiffness" of the differential equations involved. It has been clearly documented by many previous studies (see Ref. 4, for example) that attempts to solve the interaction problem with forward marching techniques were plagued by the "branching" phenomenon wherein small changes at the initial plane produced enormous changes at the exit plane. Clearly, then, small downstream changes should produce infinitesimal changes at the initial plane, as is apparently the case depicted in Fig. 4. It would thus appear that while the exit plane boundary condition is an issue that needs clarification, it should have little impact on the study of the flow far forward of the exit plane.

One final point should be made concerning these constant blowing rate solutions. It was pointed out by Smith and Stewartson,[4] and is equally apparent from Fig. 2 that discontinuities occur in these flows both at the onset and at the end of the injection region due to the discontinuous nature of the boundary condition on V_w. While account was made of this in Ref. 4 through use of a multileveled finite-difference grid system in this region, no attempt was made here to accommodate this difficulty. While this certainly introduces a level of numerical error in the results, it is clear from the comparisons of Fig. 3 that the qualitative nature of the solution has apparently been captured by the present results. While inclusion of a mechanism for properly representing the discontinuity could be pursued following the approach used by Srivastava et al.[23] for a similar difficulty in viscous shock-layer theory, this did not seem warranted here since enough points of interest could be made without this refinement. In the next flow configuration studied, the issue was sidestepped completely by employing wall injection velocity distributions that were continuous throughout the region of interest.

The next configuration studied was that of flow with a nonuniform injection velocity distribution with a continuous V_w distribution that attains a peak and thereafter drops over the same region previously considered. Study of such cases affords the opportunity to test further the applicability of the numerical scheme, to assess the influence of variable V_w on the flow structure, and to compare results of the present study with those related to ablation-generated injection where the blowing rate is expected to vary with distance along the surface. In order to make these comparisons, the same basic flat plate flow just studied was again considered with the blowing distribution modified significantly. As shown in the lower part of Fig. 5, it was first assumed that a reduced level of injection existed all the way from the leading edge up to the point $s=1.0$. Beyond this point a rapid increase in injection was specified, peaking at $s=1.6$ and decreasing to a reduced level at $s=2.2$. In particular, it was assumed that

$$V_w=0.1, \quad 0\leq s\leq 1 \tag{15a}$$

and

$$V_w=V_{w_{\max}}-(V_{w_{\max}}-0.1)[(s-1.6)/0.6]^2, \quad 1\leq s\leq 2.2 \tag{15b}$$

where $V_{\max}$ was varied from 0.1 to 4.0. (For self-similar flow, blowoff will occur at $V_w=0.88$.)

In order to accommodate the effect of injection ahead of the computational grid ($s=0.4$), the influence of the upstream blowing on the weak interaction solution (imposed at $s=0.4$) has to be established. This is accomplished using a local similarity approach where it is assumed that the ξ derivatives in Eqs. (2a-2c) are small, thereby reducing this set to ordinary differential equations. In order to estimate the edge properties β and U_e^2/T_e appearing in these equations, a weak interaction concept is employed. Thus their values were obtained using the noninteraction solutions on a flat plate with blowing to determine the boundary-layer growth from Eqs. (7a) and (10). This, in turn, provides the necessary estimate to the flow deflection angle to be used in the tangent pressure law to estimate β and U_e^2/T_e. Thus a completely consistent initialization procedure can be easily carried out for this flow configuration. The results obtained inherently reproduce the weak interaction solutions recently presented by Inger and Swean.[24]

As shown in Fig. 5, downstream, where the blowing rate increases rapidly, there is a rapid increase in the displacement body height Δ, producing the pressure peak reminiscent of the uniform blowing case. In the present case, however, the rather smooth increase in V_w removes the sharp gradients observed in the pressure distributions near $s=1.0$ with uniform injection. The skin friction shows a marked reduction in the injection region but a relatively rapid recovery toward the weak interaction level ($V_w=0.1$) as V_w drops off smoothly downstream. As the blowing rate is increased to its maximum level, separation is again observed to first occur ahead of $s=1.0$ with reattachment forced by the large injection rate. Again, it is important to realize that this return to a positive shear level in the strong blowing region is not "reattachment" in the usual sense. It should be apparent that the boundary layer that lifted off the surface does not return to the surface at this point. Rather, the zero shear point here is more indicative of mere alignment of the injected fluid than of reattachment. Thus ahead of $s=1.12$, the injected fluid is actually being drawn forward along the plate surface toward the leading edge before being lifted up and swept downstream. All of the injected fluid aft of $s=1.12$ is immediately being swept aft along the plate, giving rise to the low but nonzero shear in the downstream region. As the injection velocity drops, entrainment takes place and the shear level ultimately increases to its undisturbed level.

Conclusions and Recommendations

Based on the results achieved in this study, it appears reasonable to conclude that the numerical technique of Werle and Vatsa[5] is well suited to solving the interaction-injection problem. Limited but favorable comparisons with Stewartson[2] and Smith and Stewartson's[4] triple-deck formulation give credence to the interacting boundary-layer formulation of this problem. It was found that Stewartson and Smith's conjecture of a pressure rise and separation bubble emergence ahead of the slot followed by a continual pressure drop along the slot length was verified in all cases studied here.

Future studies of the slot injection problem should be conducted over a wide range of flow parameters. Correlation of these studies should be attempted using the triple-deck analysis to identify the scaling laws and thus correlation parameters. Correlation attempts should then be applied to appropriate experimental data to test the limitations of the approach.

The numerical method applied in the slot injection study should be adjusted for the constant injection velocity case in order to properly represent the pressure gradient discontinuity at the slot terminal points. Such techniques have alreay been developed and applied in Ref. 23 for numerically accommodating the pressure gradient discontinuity occurring at a point of surface curvature discontinuity.

In addition, future studies should also be directed toward application of the present technique to the problem of plate injection into a supersonic stream. Here interest should center on determining if and when blowoff (catastrophic separation) will occur (as per the discussion of Stewartson[2]) once interaction effects are accommodated. Preliminary calculations for this problem were conducted in this study indicating that interaction effects either eliminate or significantly delay the occurrence of blowoff. However, difficulties occur with the application of the current numerical algorithm to this problem. Far downstream in plate blowing a large region of essentially inviscid flow develops above the plate surface under the viscous boundary-layer region. This convection-dominated region is not well represented by the current diffusion-type boundary-layer solver. Attempts to correct this deficiency, perhaps along the lines presented in Ref. 25, should be initiated.

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